REVIEWS

Principles of Classical Mechanics and Field Theory. (Vol. III, Part 1, of *Encyclopedia of Physics*, edited by S. Flügge.) Springer, 1960. 902 pp. DM. 198.

(This volume contains the following articles:

Classical dynamics, by J. L. Synge, pp. 1–225.

The classical field theories by C. Truesdell and R. A. Toupin, pp. 226–793.

Appendix—*Tensor fields*, by J. L. Eriksen, pp. 794–858.

Only the second comes within the scope of this review.)

Truesdell and Toupin's monumental treatise is devoted to a '*formally precise*'† study of the general equations governing the classical field theories of continuum mechanics. Seeking the greatest possible generality and only occasionally proceeding from the general to the particular, it is rather more in the Continental than the British tradition. Thus, in both conception and spirit, it is poles apart from Rayleigh's classic *Theory of Sound*,‡ although perhaps closer to Kelvin and Tait's *Treatise on Natural Philosophy*. To illustrate these remarks by direct quotation, before getting down to technical content, seems both informative and pleasurable.

Regarding the nature and plan of the treatise:

> We present the *common foundation* of the field viewpoint. We aim to provide the reader with a full panoply of *tools of research*, whereby he himself, put into possession not only of the latest discoveries but also of the profound but all too often forgotten achievements of previous generations, may set to work as a theorist.
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> This treatise is intended for the specialist, not the beginner. Necessarily it presents the foundations of the field theories, not as they appeared in the last century and linger on in the textbooks, nor as the experts in some other domains may think they ought to be presented, but as they are cultivated by the specialists of today....
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> That over one half of the work is devoted to kinematics, the mathematical description of motion, is not malapropos. As the need for more and more general field theories has grown, the preliminary light which kinematics unencumbered by physical restrictions can provide, always appreciated by virtuosi of mechanics, has become a necessity. [Kelvin and Tait also devoted over one half (excluding the chapter on measurements) of their treatise to kinematics.] In presenting here as our Chaps. B and C the first general treatise on the kinematics of continua, we believe that we look towards the future course of the field theories.

Regarding tradition:

> We have tried to supply full and correct attributions, not only for historical perspective but also in plain justice. If the name attached to many a proposition is

† Italics denote authors' emphasis in this and all subsequent quotations. Material in square brackets has been interpolated by the reviewer.

‡ It is perhaps significant that Rayleigh's name does not appear in the index of the present treatise, whereas there are fourteen entries under Euler. It also may interest the typical reader of this review to note that only one of the eighty-four papers cited in the period 1956–60 appeared in the *Journal of Fluid Mechanics*.

but a small one, that is all the less reason that its owner should be pilled of what little he wrought by a no greater name of today, whose slight capacities are scarcely increased by wilful or heedless ignorance of what others have done. However, the multitude of detailed citations should not prevent the great names from emerging. Our subject is largely the creation of EULER and CAUCHY. If we present their results in forms often very different from the original, in return we have included many of their discoveries that have not previously found a place in expositions. Not only will these names be the most frequently encountered, but also their appearances are at the crucial theorems and definitions. Next come STOKES, HELMHOLTZ, KIRCHHOFF, KELVIN, MAXWELL and HUGONIOT. In the twentieth century, HADAMARD and HILBERT continued and deepened the tradition. That no one later name is frequently cited does not indicate that the subject is dead. Rather, after a generation of quiescence, in very recent years it has experienced a revival in a form more compact and general, and, we believe, closer to nature....

Any partial bibliography of work on the concepts and axioms of mechanics from the origins through the time of LAGRANGE would be misleading. No adequate critical history has ever been written. The remarks on this subject given in treatises or general histories of physics are often mendacious and usually so incomplete and inaccurate as to be totally misinformative.

Regarding the foundations of mechanics:

... *Euler's laws* of mechanics; these integral relations express for an entire body, whether solid, superficial, lineal, or punctual in form, the balance of momentum and of moment of momentum..

....

It should be, but unfortunately it is not, unnecessary to comment that the laws of NEWTON [as stated in the *Principia* (1687)] are neither unequivocally stated nor sufficiently general to serve as a foundation for continuum mechanics...........

....

It is obvious that the converse problem of derivation of EULER'S laws...from NEWTON'S laws cannot even be stated meaningfully without severe restrictions. Nevertheless, the mass-point is so ingrained in paedogogical repetition that even today many textbooks strive to foster an illusion that NEWTON'S laws suffice as a basis for mechanics.

Regarding experiments:

The field, infinite in extent and indefinitely divisible, is by its very nature not measurable directly. The 'experiments' sometimes used as the starting point for paedagogical treatments of field theories are *a posteriori* verifications at best; always unperformed and often unperformable, too often they are mere hoaxes. Moreover, they belie the true course by which the field theories have developed. *Experience* has been the guide, *thought* has been the creator.

Regarding mathematics and its physical interpretation:

The developments must illumine the *physical aspects* of the theory, not necessarily in the narrower sense of prediction of numerical results for comparison with experimental measurement, but rather for the grasp and picture of the theory in relation to experience. In this spirit do we pursue our subject, *neither seeking nor avoiding* mathematical complexity.

Compare Kelvin and Tait: 'The *Laws of Motion*, the *Law of Gravitation and of Electric and Magnetic Attractions, Hooke's Law,* and other fundamental principles derived directly from experiment, lead by mathematical processes to interesting and useful results, for the full testing of which our most delicate experimental methods are as yet totally insufficient.' But Kelvin and Tait follow with the *caveat*, 'Nothing can be more fatal to progress than a too confident reliance on mathematical symbols; for the [reader] is only too apt to take the easier course, and consider the *formula* and not the *fact* as the physical reality.'

Regarding mathematical rigor:

> *In physical theory, mathematical rigor is of the essence.*

Compare Rayleigh: 'In the mathematical investigations I have usually employed such methods as present themselves naturally to a physicist.... To his mind, exercised in a different order of ideas, the more severe procedure of the pure mathematician may appear not more but less demonstrative.'

Regarding closed systems:

> For example, it is often claimed that in nature, if we look closely enough, only conservative forces occur; that such effects as friction are gross appearances resulting only from lack of knowledge of the underlying conservative process. But natural problems are not confined to those on the smallest or largest scale. The world about us, as we see it, must be mastered and controlled. Situations incompletely described are the rule, not the exception, and we must formulate good theories for these *limited aspects* of nature. Our object is a *general framework* for such theories. The most general motions, the most general stresses, the most general flows of energy, and the most general electromagnetic fields, furnish the subject of this treatise.

Regarding materials:

> The present treatise is devoted to the *general principles of balance* alone. Thus we deal only with the *field equations and jump conditions*. Our last chapter mentions guiding principles by which rational constitutive equations may be formulated. The theories of certain particular ideal materials fill several later volumes of the Encyclopedia.

The treatise is divided into seven chapters. Chapter A, *The field viewpoint in classical physics* (15 pp.), is introductory and describes the authors' general approach (see the foregoing quotations).

Chapter B, *Motion and mass* (251 pp.), deals with kinematics, including conservation of mass, and 'collects and organizes all the researches [that the authors] have been able to find on the kinematics of continuous media'. It is subdivided into three parts. Part I, *Deformation*, deals with the theory of a single deformation and contains subsections on deformation gradients, strain, rotation, special deformations, small deformation, and 'oriented bodies' ('apparatus for a type of physical theory as yet little studied but likely to be of future value'); it is likely to be of greater interest for what is conventionally regarded as solid mechanics than for fluid mechanics. Of greater interest for fluid mechanics is Part II, *Motion*, which deals with continuous motions and contains subsections on velocity, material systems, stretching and spin, acceleration, vorticity, 'further special motions' (e.g., 'homogeneous motion', analogous to homogeneous strain), and relative motion. As might have been anticipated from Truesdell's numerous contributions to the literature, the section on vorticity is a monograph in itself; it contains such iconoclastic disclosures as 'HELMHOLTZ erroneously concluded that vortex lines either form closed curves or else end on a boundary'. (Compare Lamb's *Hydrodynamics*, §145: 'Any vortex-lines which exist must either form closed curves, or else traverse the fluid, beginning and ending on its boundaries.') Part III, *Mass*, treats 'topics in kinematics whose usefulness is connected with mass' and contains subsections on the definition of mass, the solution of the equation of continuity, and momentum.

Chapter C, *Singular surfaces and waves* (40 pp.), organizes and describes 'those properties of surfaces of discontinuity, such as vortex sheets, shock waves, and acceleration waves, as are common to all media'. It contains five parts on the geometry of singular surfaces, the motion of surfaces, kinematics of singular surfaces, singular surfaces associated with a motion, and discontinuous equations of balance. The reader is warned that:

> In works on physics we often encounter discontinuous solutions regarded as limits of continuous ones....In gas dynamics, a flow of an ideal gas with a shock wave is regarded as the limit of solutions of a corresponding boundary value problem for viscous, thermally conducting fluids as the viscosity and thermal conductivity tend to zero. Problems of this kind presuppose some definite theory of materials and have no meaning in the generality of the present treatise.

Chapter D, *Stress* (77 pp.), 'presents the laws of classical mechanics and the general theory of contact forces or stress, in terms of which mechanical theories of continuous media are formulated'. It contains five parts on the balance of momentum, the stress principle, application of Cauchy's laws, general solutions of the equations of motion, and variational principles. The authors claim that: 'While every textbook on elasticity or plasticity makes some show of presenting the general theory of stress, no reasonably comprehensive exposition has ever been published before.'

Chapter E, *Energy and entropy* (53 pp.), presents 'the general theory of energy in unexceptional terms [and] the more dubious subject of thermodynamics set within field concepts'. It is divided into three parts on the balance of energy, entropy, and equilibrium. The authors claim that: 'There exists no other comparably general and complete exposition of the material given in this chapter.'

Chapter F, *Charge and magnetic flux* (40 pp.), adds the principles of conservation of charge and of magnetic flux to the principles of classical mechanics and departs from the three-dimensional, Euclidean space of the preceding chapters by introducing a world-invariant formulation in four-dimensional space-time.

> The sequence of hypotheses and the order of logical development...depart from the traditional treatments....For example, we regard the conservation of charge as a physical or intuitive concept logically independent of the concepts of rigid rods, uniform clocks, and inertial frames, and we have chosen to express this law in a mathematical form likewise independent of the representation of these extraneous entities.

Chapter G, *Constitutive equations* (45 pp.), is intended to be selective and illustrative, rather than definitive. It is divided into one part on generalities and six parts on kinematical, energetic, mechanical, thermo-mechanical, electromagnetic, and electromechanical constitutive equations. Referring to the simple divisions of phenomena on which the classical theories of physics are based, the authors regard the separation as 'unnatural and unjustified' and introduce the

> **principle of equipresence:** *A variable present as an independent variable in one constitutive equation should be so present in all*....It may be regarded as a natural extension of OCKHAM's razor as restated by NEWTON: 'We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances, for nature is simple and effects not the pomp of superfluous causes'.

Sections 297–299 contain a compact and elegant discussion of the constitutive equations for fluids and illustrate the clarity to be gained by eschewing the generality of preceding chapters.

The *List of works cited* (40 pp.) covers the period 1678–1960, beginning with Hooke and ending with Truesdell. Also included are four additional bibliographies (9 pp.) on kinematics of special motions, non-relativistic kinematics and mechanics in generalized spaces, principles of mechanics, and relativistic continuum theories.

In writing this review, I have preferred to let the authors speak for themselves wherever possible, believing that some of both the merits and the demerits of their presentation are implicit in the foregoing quotations. Nevertheless, there is one flaw that appears to demand explicit mention, and I must join with Professor Prager (*Mathematical Reviews*, **22**, 1961, 1494) in deploring the authors' predilection for the greatest possible generality. Among other drawbacks, this quest of generality leads to an appallingly complicated notation, based on Roman, italic, bold-face, Hebrew, Russian, script, German, Greek, and sans-serif letters among the full-sized characters and Roman, italic, Greek, bold-face, and German indices; even these prove insufficient, and Roman numerals are brought in to denote principal invariants and moments of tensors. A *partial* table of symbols occupies four pages, but the authors still find it necessary to apologize that 'it has not been possible to maintain this scheme without exception, nor to avoid use of the same symbols in different senses in widely separated passages'. The authors claim that 'a reader with some experience in the subject will be able to start at any point he pleases', but the implications of 'some' can only be regarded with dismay.

A truly definitive review of this impressive work would have been beyond my competence. Still, I should be remiss were I not to conclude by recording my opinion that we have here a classic in the first sense of the word. In scholarship, in outlook, and in command of the English language, it is surely unique in our time, and there is every reason to believe that it will be the preeminent treatise in its domain for many decades to come.

JOHN W. MILES

An Elementary Treatise on the Mechanics of Fluids. By W. J. Duncan, A. S. Thom and A. D. Young. London: Edward Arnold, 1961. 714 pp. 70*s*.

The authors state in the preface, 'This book is intended primarily for the use of undergraduates and students in Technical Colleges but it is hoped that it may be of service to young professional engineers and physicists and also to independent and unassisted readers'. They go on to say that their aim is to provide a systematic and easily understood account of the basic principles of the science of the mechanics of fluids.

The book begins with a discussion of the properties of fluids and in chapters 2 and 3 treats 'classical' fluid mechanics, going quite a long way with kinematics before beginning dynamics in chapter 3. The treatment of the stream function and velocity potential is conventional, but it is of interest that the complex potential is introduced by considering the wave equation.

Students often have difficulty with dimensional analysis, and so it is encouraging to find that the short treatment in chapter 4 is very good, following the lines of the book on this subject by Prof. Duncan. Chapter 5 is a short survey of experimental techniques.

Chapter 6 is the longest and the best chapter in the book. In 150 pages it introduces the reader to the physics of boundary layers and leads him gently through the calculation of laminar and turbulent boundary layers. Boundary-layer control is discussed and some attention is given to stability problems. This chapter also gives numerous references to original papers.

This is followed by two chapters on the flow in pipes and open channels and the reader is immediately brought back to the more empirical approach of engineering hydraulics. These chapters treat the usual notches, weirs and orifices, and one wonders if it is worth deriving the equations for the flow of a perfect fluid through these constrictions in such detail.

The next chapter introduces compressible flow and some time is spent on elementary thermodynamic concepts which are neglected in some books on fluid mechanics. This useful chapter touches upon the linearized theories of subsonic and supersonic flow, and includes physical descriptions of transonic and hypersonic flow.

Chapter 10 deals with oscillations and waves, and chapter 11 with the forces and moments on bodies in a stream. This gives an outline of aerofoil and wing theory, including slender-wing theory. Also touched upon are aircraft stability derivatives and the flow in cascades. It is slightly annoying to find some figures in this chapter showing flow from right to left while other closely related figures show the flow in the opposite direction.

The last two chapters consider pumps and turbines and fans and propellers.

Does the book achieve the aims outlined in the preface? On the whole I think that it does, the chapters on classical fluid mechanics and boundary layers being very good and the other topics being adequately treated. The book is very readable and contains a large number of examples and problems. It should therefore be useful to undergraduates although it may well be too expensive for them.

D. J. Maull

Atmospheric Diffusion. By F. Pasquill. Van Nostrand Co., 1961. 297 pp. 60*s*.

Discussion of matter in a turbulent flow is a problem of special interest for students of turbulence. Historically, the paper by G. I. Taylor on diffusion by continuous movements was the beginning of the statistical theory of turbulence which has been the dominant influence in the subject. Although mathematical difficulties have prevented a full development of the theory except for the special case of homogeneous turbulence, the basic view that a thorough understanding of the velocity fluctuations is essential to an understanding of the properties of the mean flow has provoked many experimental studies of the fluctuations and has established new standards for the acceptance of phenomenonological theories. Its influence on meteorological study of the lower atmosphere has been considerable, perhaps because the scale of the fluctuations is such that it is difficult to ignore them, and persistent attempts have been made to use the theory in a practical way. In the earth's boundary layer, the steady-state transport of momentum, sensible heat and water-vapour can be described adequately by the mixture of statistical theory, empirical generalization and convenient assumption that has developed in the last ten years, but transient diffusion is a more difficult problem. The reason is that convection of matter is described most simply in the Lagrangian specification of the fluid motion which represents the velocities of individual fluid particles as functions of time and their initial positions. With few exceptions, turbulent velocities are measured at points fixed in space and so refer to the Eulerian specification of the velocity field as a function of time. For theoretical purposes also, the Eulerian specification is relatively more convenient, and there is a considerable gap between the 'Eulerian' theory of turbulence, based on considerations of statistical equilibrium, and the theory of diffusion by continuous movements.

Dr Pasquill's book might be regarded as an account of attempts to establish methods of predicting diffusion in the atmosphere which use practicable measurements of velocity and temperature and which are broadly consistent with current ideas of the nature of turbulent motion. The attitude of the author is made clear by the first sentence, which introduces the velocity fluctuation, postponing consideration of diffusion until the statistical techniques for specifying a random velocity field have been described and until observations of intensities and spectra have been considered. Dr Pasquill has been very active in this field and he writes with an awareness of the difficulties of obtaining significant information from records whose duration is comparable with the periods of some components and which have been obtained with instruments of finite resolution. Besides laying a foundation for the later chapters, the section on intensities and spectra is a very useful summary of material not easily accessible, particularly to a non-meteorologist. The chapter on diffusion introduces the two theoretical frameworks for the analysis of atmospheric diffusion, one involving use of the diffusion equation with an eddy diffusivity varying with height and the other use of the statistical theory of diffusion by continuous movements with the appropriate Lagrangian correlation functions. The considerable differences between time-mean diffusion from a maintained source and diffusion of a cluster are brought out and related to the characteristic actions of eddies of different sizes. Experimental studies of diffusion are described and correlated with measurements of turbulent intensity and scale. Except in very stable conditions, the correlation is generally satisfactory. The last chapters deal with the estimation of diffusion for the practical purpose of the operational meteorologist and with consideration of other kinds of diffusion dependent on the large-scale motion of the atmosphere, e.g. the distribution of radioactive material from tests of nuclear bombs.

The plan and balance seem to me very good and the book should be useful to many species of physicist and applied mathematician. As a book written with a practical object, it is a welcome contrast to the typical text-book on fluid mechanics which deals with turbulent flow by appending to a severely utilitarian account of friction coefficients a brief and irrelevant account of isotropic turbulence. The inclusion of so much basic theoretical and experimental material in a book of only 274 pages means that the facts-per-page ratio is high and that it needs attentive reading, but I regard it as essential reading for anyone interested in atmospheric turbulence in general and not just in atmospheric diffusion.

A. A. Townsend